



LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

FIRST SEMESTER – NOVEMBER 2011

MT 1501 - GRAPHS, DIFF. EQU., MATRICES & FOURIER SERIES

Date : 10-11-2011
Time : 1:00 - 4:00

Dept. No.

Max. : 100 Marks

Part A (10 x 2 = 20)

Answer All questions.

1. Find the equation of the line passing through (2, 9) and (2, -9).
2. Find the axis and vertex of the parabola $y = x^2 + 2x + 3$.
3. Write the normal equations of $y = ax + b$.
4. Define linear law.
5. Form the difference equation of lowest order by eliminating the arbitrary constants a and b from $y = (a + bx) 2^x$.
6. Solve $y_{n+2} - y_{n+1} + y_n = 0$.
7. Define symmetric and skew-symmetric matrices.
8. Find the eigen values of the matrix $\begin{pmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$.
9. Find the Fourier coefficient a_n for the function $f(x) = e^x$ in $(-\pi, \pi)$
10. Define odd and even functions.

Part B (5 x 8 = 40)

Answer any Five questions.

11. The total cost (in rupees) of output x is given by $C = \frac{2}{3}x + \frac{35}{2}$. Find
 - (i) The cost when the output is 4 units.
 - (ii) The average cost of output of 10 units.
 - (iii) The marginal cost when the output is 3 units.
12. A firm produces x units of output per week at a total cost of Rs. $\frac{1}{3}x^3 - x^2 + 5x + 3$. Find the value of x at which the marginal cost and the average cost attain their respective minimum.
13. Using the method of least squares, fit a straight line to the following data.

x	5	10	15	20	25
y	15	10	20	26	30

14. Explain the method of least squares.

15. Solve the difference equation $y_{n+2} - 2y_n \cos \alpha + y_{n-1} = 0$. If $y_0 = 0$ and $y_1 = 1$, Find y_2, y_3, y_4 .

16. Find the eigen values and eigen vectors of $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 2 \end{pmatrix}$

17. Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$

18. Obtain the Fourier expansion for $f(x) = x - \pi$ in the interval $(-\pi, \pi)$

Part C (2 x 20 = 40)

Answer any Two questions.

19. (a) From the table given below, fit an equation of the form $y = a + bx + cx^2$.

x	87.5	84	77.8	63.7	46.7	36.9
y	292	283	270	235	197	181

(b) The price and demand for an item are related by $p = 32 - x^2$, while price and supply are related by $p = x^2$. Draw the graph and find the equilibrium supply and equilibrium price.

(12 + 8)

20. Solve the difference equations:

(a) $y_{x+2} - 5y_{x+1} + 6y_x = x^2 + x + 1$

(b) $u_{n+2} - 7u_{n+1} - 8u_n = 2^n n^2$

(10 + 10)

21. (a) Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$ and deduce that the sum

of the series $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

(b) Find a Fourier series expansion for the function $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \leq x \leq \pi \end{cases}$

(10 + 10)

22. Diagonalize the matrix $\begin{pmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$.
